Half Yearly Examination-2020-21

B.C.A. Part-III

PAPER-FIRST: CALCULUS & GEOMETRY

[Time- 3 hours] [Maximum Marks : 50]

Note: Attempt any two parts from each question, All questions carry equal marks.

1.(a) Let F ∈ R [a,b] & Let F be a differentable on [a,b] such that f'(x)= f(x), x϶[a,b] then show that $\int_a^b f(x)dx$= F(b)-F(a)

(b) Let f(x) = x on [0,1] then show that f is R-integrable on [0,1] and $\int_a^b f(x)dx=\frac{1}{2}$

(c) If f ∈ R [a,b] & m,M are respectively infirm and supremum of function f over [a,b] , then prove that

$$m(b-a) \leq \int_a^b f(x)\, dx \leq M(b-a) \quad (a \leq b)$$

2.(a) Examine for maxima and minima of the function

$$f(x,y) = x^3-4xy+2y^2$$

(b) Find for minimum value of $u = x^2 + y^2 + z^2$ when ax + by + cz=P

(c) Find the maxima & minima of u= Sin A , Sin B , Sin C

If A, B, and C are the Angles of the ΔABC.

3.(a) prove that integral $\int_0^\infty dx/(x-a)(\sqrt{b}-x)$ is divergent.

(b) test the convergence of the integral. $\int_{-\infty}^{\infty} \frac{dx}{x(1+x^2)}$

(c) Discuss the convergence of gamma function $\int_0^\infty x^{n-1}e^{-x}$ dx .

4.(a) find the eqation of the cone whose vertex is (a,b,c) & base curve $(x^2/a^2) + (y^2/b^2) = 1$,

Z = 0.

(b) prove that the equation of the right circular cone whose vertex is the origin axis is z-axis and semi vertical angle is α is $x^2 + y^2 = z^2 \tan^2 \alpha$.

(c) Prove that the equation

$$ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$$

represent a cone of $\frac{u2}{a} + \frac{v2}{b} + \frac{w2}{c} = d$

5.(a) explain relation between Cartesian &polar co-ordinates.

(b) to find the polar equation of a conic with its latus rectum of length 2l,eccentricity e & the focus being the pole.

5.(c) to find the polar equation of the chord joining two points 'α' and 'β' on the conic l/r = 1 + ecosθ.

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